Longest Common Subsequence (and an ILP)

A subsequence

For a string *S* a **subsequence** is a subset of characters from the string in the same relative order.

Formally: a subsequence is specified by a set of indices $i_1 < i_2 < \ldots < i_k$, such that $k \leq m$. The subsequence is then $S[i_1] \cdot S[i_2] \cdot S[i_3] \cdot \ldots \cdot S[i_k]$.

The LCS Problem

Given

- two strings S and T

Find

- the subsequences $\hat{S} = \hat{T}$
- of maximum length

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$S = ATTCGATACAGTG$

$T = TATCTGAGGTGA$

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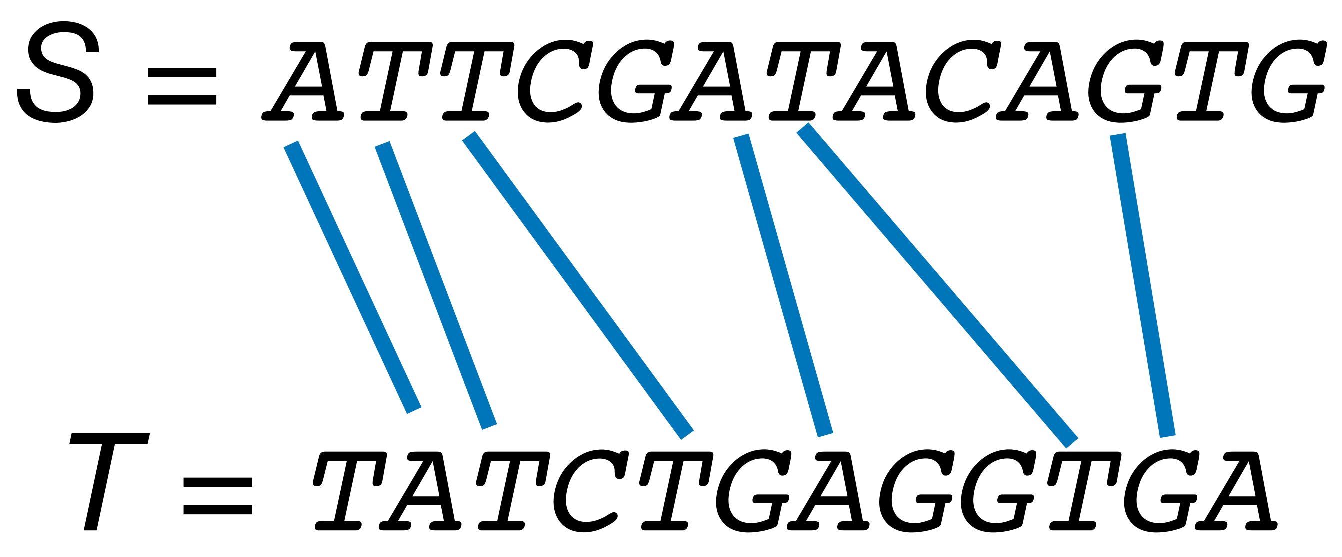

ATTATG

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ATTATG

ATTGATG

A Dynamic Program

L[0][j] = L[i][0] = 0

$$L[i][j] = \max \begin{cases} L[i-1][j-1] + 1, & \text{if } S[i] == T[j] \\ L[i-1][j] \\ L[i][j-1] \end{cases}$$

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What about as an ILP?

An ILP

maximize $\sum_{1 \leq i \leq n} \sum_{1 \leq j \leq m} P(i,j)$

subject to

$$\sum_{1 \leq j \leq m} P(i,j) \leq 1 \qquad \forall i$$

$$\sum_{1 \leq i \leq n} P(i,j) \leq 1 \qquad \forall j$$

$$P(i,j) + P(i',j') \leq 1 \qquad i < i' \,\&\, j' < j$$

$$P(i,j) \in 1,0 \qquad \forall i,j$$